DSII: Quiz 6

Name:	**ID:**

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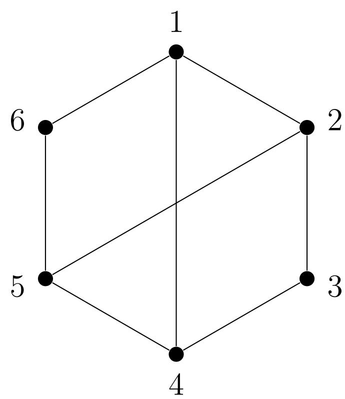
1. Consider the graph G shown below.



Find the degree of each vertex. For each vertex, list its neighbors explicitly, then state its degree.

Solution:

2. Using the same graph G from Question 1:

(a) Find $N(1)$ and $N(3)$, where $N(v)$ denotes the *neighborhood* of v, the set of all vertices directly connected to v by an edge.

(b) Verify the Handshake Theorem.

Solution:

3. State the condition that an undirected, connected graph must satisfy in order to have an **Euler circuit**.

Solution:

4. Consider the graph with vertex set $\{A, B, C, D\}$ and edge set

$$\{\{A, B\},\ \{B, C\},\ \{C, D\},\ \{D, A\},\ \{A, C\},\ \{B, D\}\}.$$

(a) Draw this graph in the solution box below.

(b) Is this graph *simple*?

Solution:

5. A connected planar graph has $v = 8$ vertices and $e = 13$ edges.

(a) Use Euler's formula to find r.

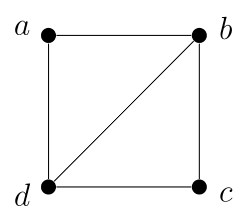
(b) Verify that this graph satisfies the general planar graph inequality. Show your calculation and state whether the inequality holds.

Solution:

6. A graph has degree sequence $4, 4, 3, 3, 2, 2, 1, 1$.
Use the Handshake Theorem to determine the number of edges $|E|$. Show all steps clearly.

Solution:

7. The graph below has vertices a, b, c, d.



Find the degree of every vertex. Then determine whether the graph has an **Euler circuit**, an **Euler trail** (but not a circuit), or **neither**.

Solution:

8. The *wheel graph* W_5 has vertex set $\{0, 1, 2, 3, 4, 5\}$ and edge set:

$$\{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 1\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{0, 4\}, \{0, 5\}\}.$$

Find the chromatic number $\chi(W_5)$, the minimum number of colors needed so that no two adjacent vertices share the same color. Your solution must include:

(a) An explicit valid coloring of all 6 vertices using the minimum number of colors. Label every vertex with its assigned color.

(b) A clear argument for why one fewer color is not sufficient.

Solution:

9. A graph has a self-loop at vertex 1 and also has three parallel edges connecting vertices 2 and 3. Is this graph simple? Justify your answer.

Solution:

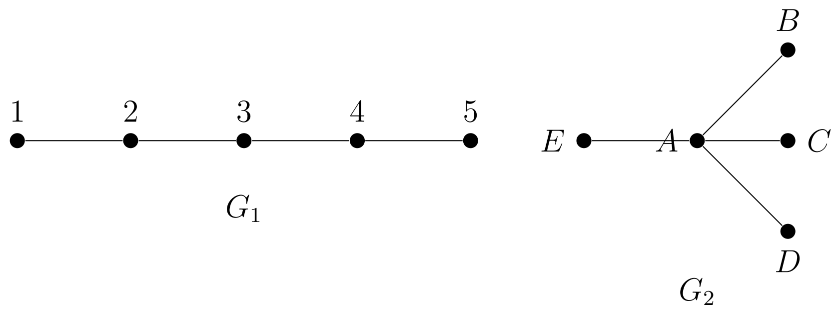
10. The complete bipartite graph $K_{2,3}$ has two disjoint vertex sets $\{u_1, u_2\}$ and $\{v_1, v_2, v_3\}$. Every vertex in the first set is connected to every vertex in the second set; no edges exist within either set.

(a) How many edges does $K_{2,3}$ have? Show your calculation.

(b) Find the chromatic number $\chi(K_{2,3})$, the minimum number of colors needed so that no two adjacent vertices share the same color. Justify your answer: explain why one fewer color is insufficient and provide an explicit valid coloring.

Solution:

11. The two graphs G_1 and G_2 below each have 5 vertices and 4 edges.



(a) Find the degree sequence of each graph and write their the full degree sequence in decreasing order.

(b) Determine whether G_1 and G_2 are isomorphic. If they are *not* isomorphic, state a specific invariant (a property that must match for isomorphic graphs) that differs between them.

Solution:

12. Explain the difference between an **Euler path** and a **Hamiltonian path**.
In your answer, clearly state:

- What each path traverses (edges or vertices).
- Whether edges or vertices may be repeated.

Solution:

13. A graph has degree sequence $(5,\ 4,\ 3,\ x,\ 2,\ 2,\ 1,\ 1)$ and has exactly 10 edges.
Use the Handshake Theorem to find the value of x. Show all steps.

Solution:

14. A connected planar graph has $v = 6$ vertices and $e = 9$ edges.

(a) Use Euler's formula to find r, the number of regions the graph divides the plane into, including the unbounded outer region.

(b) Apply the general planar inequality. Show your calculation and state whether the inequality holds.

Solution:

15. Let

$$G:\ V=\{1,2,3,4,5\},\quad E=\big\{\{1,2\},\{2,3\},\{3,4\},\{4,5\},\{1,5\},\{1,3\}\big\}$$

and

$$H:\ V=\{A,B,C,D,E\},\quad E=\big\{\{A,B\},\{B,C\},\{C,D\},\{D,E\},\{A,E\},\{B,D\}\big\}.$$

(a) Find the degree of each vertex in G and in H. Write the degree sequence of each graph in non-increasing order.

(b) Determine whether G and H are isomorphic. If they are, provide an explicit vertex mapping (for example, 1→A, 2→B, ...) and verify it by listing every edge in G and confirming its corresponding edge exists in H.

Solution:

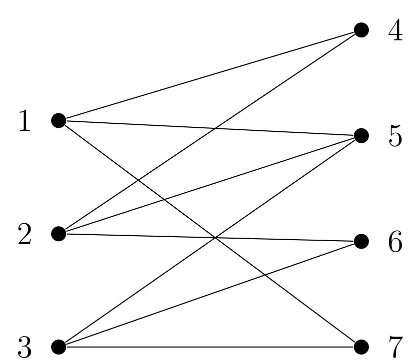
16. The complete bipartite graph $K_{3,4}$ has two disjoint vertex sets of sizes 3 and 4, with every vertex in one set connected to every vertex in the other.

(a) Find the degree of every vertex in $K_{3,4}$.

(b) Does $K_{3,4}$ have an Euler circuit, an Euler trail (but not a circuit), or neither?

Solution:

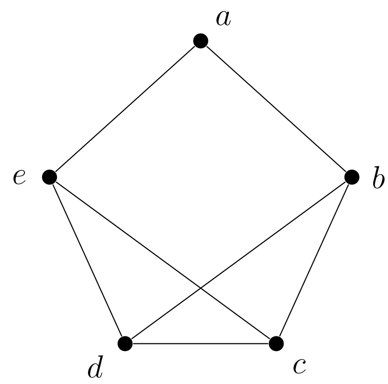
17. The graph G below has vertex set $\{1, 2, 3, 4, 5, 6, 7\}$. The vertices are split into two groups: $X = \{1, 2, 3\}$ on the left and $Y = \{4, 5, 6, 7\}$ on the right. Every edge connects a vertex in X to a vertex in Y (there are no edges within X or within Y).



Does G have a **Hamiltonian cycle**?
Your answer must include a complete justification.

Solution:

18. The graph below has vertices a, b, c, d, e.



Find the degree of every vertex. Then determine whether the graph has an **Euler circuit**, an **Euler trail** (but not a circuit), or **neither**.

Solution:

19. For each graph described below, determine whether such a graph is **possible** or **impossible**.

- If *possible*, state how many edges it has.
- If *impossible*, state a specific reason why not.

(a) A graph with 4 vertices, each of degree 4.

(b) A graph with 5 vertices, each of degree 3.

(c) A graph with 6 vertices, each of degree 4.

Solution:

20. The complete graph K_4 has 4 vertices, and every pair of vertices is connected by an edge. Answer all four parts, showing full reasoning for each.

(a) How many edges does K_4 have?

(b) Apply the general planar inequality to K_4. Show your calculation. Based on this, can you conclude that K_4 is planar, non-planar, or is more information needed?

(c) Assuming K_4 is drawn as a connected planar graph, use Euler's formula to find the number of regions r in the plane, including the unbounded outer region.

(d) Find $\chi(K_4)$. Provide an explicit valid coloring of all 4 vertices using the minimum number of colors.

Solution: