

DSII: Comprehensive Quiz

Name:

ID:

1. There are 7 days in a week. What is the minimum number of people needed to guarantee that at least two of them were born on the same day of the week? Briefly justify your answer.

Solution:

2. In a group of 40 students, 25 play soccer, 18 play basketball, and 7 play both. How many students play at least one of the two sports?

Solution:

3. What is the coefficient of x^2 in the expansion of $(1 + x)^6$?

Solution:

4. A fair six-sided die is rolled once. What is the probability of rolling a number greater than 3?

Solution:

5. Events A and B are mutually exclusive, with $P(A) = 0.20$ and $P(B) = 0.50$. Find $P(A \cup B)$.

Solution:

6. A graph has degree sequence 4, 3, 3, 2, 2. Use the Handshake Theorem to determine how many edges the graph has.

Solution:

7. State the condition on vertex degrees required for an undirected graph to have an Euler trail.

Solution:

8. The 4th row of Pascal's triangle is: 1, 4, 6, 4, 1. What is the 5th row?

Solution:

9. In a standard deck of 52 cards and 4 suits, how many cards must be picked to guarantee that at least 2 are from the same suit?

Solution:

10. A drawer contains red, blue, and green socks. You are picking socks in the dark. What is the minimum number of socks you must pick to guarantee you have a matching pair?

Solution:

11. A jar contains balls labeled 1 through 6. You draw balls one at a time without looking. What is the minimum number of balls you must draw to guarantee that at least 3 balls share the same label? Show your reasoning.

Solution:

12. In a survey of 200 people: 120 own a car, 80 own a bike, and 40 own both. A person is chosen at random. What is the conditional probability that they own a car, given that they own a bike?

Solution:

13. Find the full term (not just the coefficient) containing x^3 in the expansion of $(2x + 3)^5$. Show all steps clearly.

Solution:

14. Answer both parts below.

- (a) A store sells shirts in 5 different colors. What must the inventory of the store be in order to conclude that there are at least 3 shirts in one of the five colors?
- (b) How many people must be selected to make sure that at least 4 people are born in the same month?

Solution:

15. A single card is drawn from a standard 52-card deck. Let E be the event that the card is a Heart, and let F be the event that the card is a Face card. The deck has 13 Hearts, 12 Face cards, and 3 cards that are both. Find $P(E \cup F)$.

Solution:

16. How many integers from 1 through 100 are multiples of 2, 3, or 5? Use the Inclusion-Exclusion Principle and show all steps.

Solution:

17. The graph G has vertex set $V = \{1, 2, 3, 4, 5, 6\}$ and edge set

$$E = \{\{1, 2\}, \{2, 3\}, \{1, 5\}, \{2, 5\}, \{3, 5\}, \{4, 5\}, \{5, 6\}\}.$$

Answer all parts below.

- (a) What is the total degree of G ?
- (b) List the neighbors of vertex 2.
- (c) What is the degree of vertex 5?
- (d) Which vertices are adjacent to vertex 5?
- (e) Is K_3 a subgraph of G ? If so, name the vertices in the subgraph. If there are multiple instances of K_3 , please provide only one.

Solution:

18. Consider the following two graphs G and H :

$$G : V(G) = \{1, 2, 3, 4, 5, 6\}$$

$$E(G) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{2, 4\}, \{3, 5\}, \{4, 5\}, \{4, 6\}, \{5, 6\}\}$$

$$H : V(H) = \{A, B, C, D, E, F\}$$

$$E(H) = \{\{A, B\}, \{A, C\}, \{B, D\}, \{C, D\}, \{C, E\}, \{D, F\}, \{E, F\}, \{A, F\}\}$$

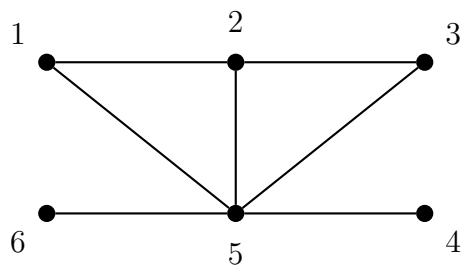
- (a) Find the degree sequence of each graph. Are the degree sequences the same?
- (b) Are G and H isomorphic? If yes, give an explicit isomorphism $f : V(G) \rightarrow V(H)$ and verify that at least two edges are preserved. If no, explain what distinguishes them.

Solution:

19. A fair coin is flipped four times. Each outcome is a string of length 4 from $\{H, T\}$, such as $HHTT$. For each event below, express the event as a set in roster notation and find its probability.
- (a) The first and last flips come up heads.
 - (b) At least two consecutive flips come up heads.
 - (c) The first flip comes up tails, and at least two consecutive flips come up heads.

Solution:

20. The graph G is depicted below.



- (a) Give the adjacency list representation of G .
- (b) Find the degree of every vertex in G .
- (c) Does G have an Euler circuit, an Euler trail, or neither? Justify your answer fully.

Solution: