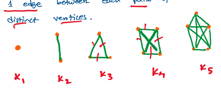
Lecture 8: Graph 2

Monday, April 27, 2026 11:06 PM

Types of Graphs

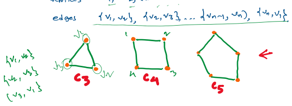
☑ Complete Graphs

A complete graph, denoted as K_n, is a simple graph that contains exactly 1 edge between each pair of n distinct vertices.

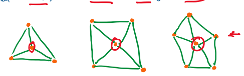


☑ Cycles and Wheels

A cycle C_n, $n \geq 3$, consists of vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_{n-1}, v_n\}, \{v_n, v_1\}$

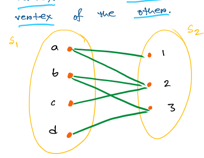


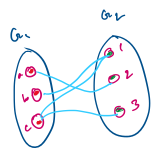
A wheel W_n is obtained when we add an additional vertex to C_n, and connect that vertex to each existing vertex.



☑ Bipartite Graphs

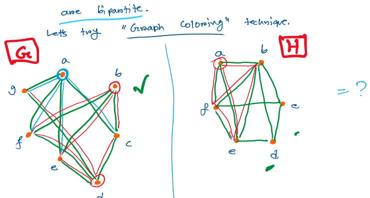
A simple graph is called bipartite if its vertex set, V, can be partitioned into two disjoint subsets such that each edge connets a vertex from one subset to a vertex of the other.

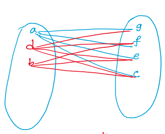




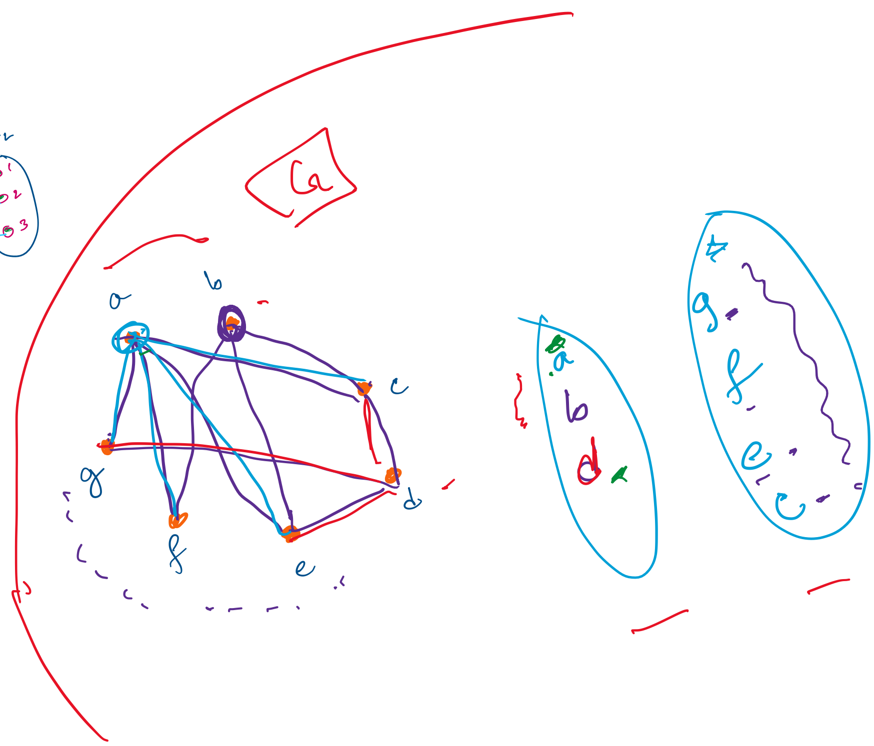
Example! Determine if graphs G or H are bipartite.

Lets try "Graph Coloring" technique.

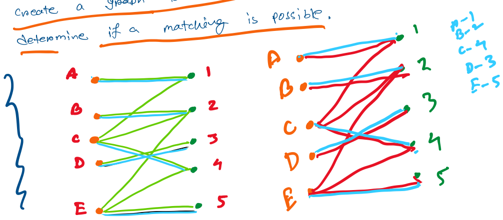




Yes G is bipartite

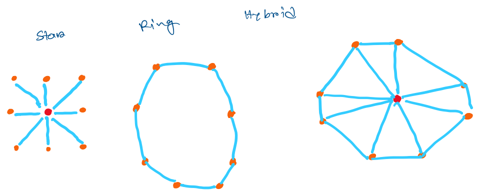


☑ Ex: suppose Adam, Ben, Chris, David, and Eric are training for tasks at work. Adam and Chris are training for task 1, Ben, Chris, and Eric are training for task 2, David is training for task 3, Chris and Eric are training for task 4, and Eric is training for task 5. create a graph to model this. then determine if a matching is possible.



Ex! suppose 8 devices (computers, printers, etc) must be connected through a local ... (LAN). Let's explore how

are network ...
this might look.



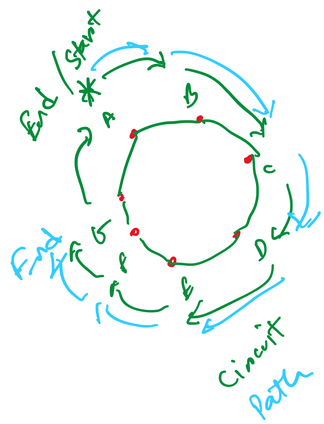
Euler Paths and Circuits

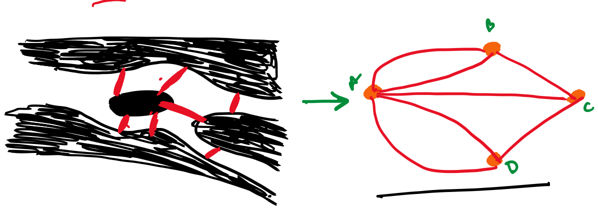
Story of Konigsberg

7 bridges of Konigsberg, which sparked the development of the entire branch of mathematics, we now call Graph Theory.

Euler wanted to determine if it was possible to start at some location in town, travel across all 7 bridge once, then return to the starting point.

This type of travel is call "Euler Circuit". It is considered a circuit because the initial vertex and the terminal vertex are the same. We can also create an Euler path, which still traverses each edge, but can begin and end at different vertices.





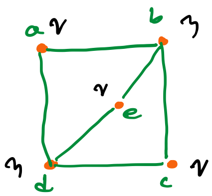
⇒ Euler Circuit – conditions – Undirected graph

G has an Euler circuit if and only if

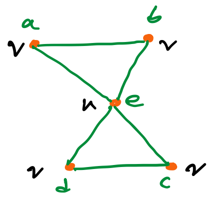
i. G is connected

ii. Every vertex has even degree

Similarly, we can say that if G is an undirected graph with no isolated vertices, then we can construct an Euler path in G if and only if G is connected and has exactly two vertices of odd degree.



G_1



G_2

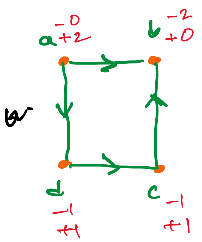
⇒ Euler Circuit - conditions - Directed Graph

G has an Euler circuit if and only if

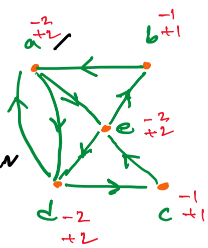
i. G is connected

ii. Every vertex has even degree

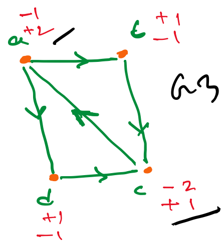
iii. $\deg^-(v) = \deg^+(v)\ \forall v \in V$



G_1



G_2

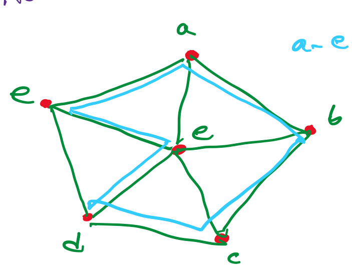


G_3

Hamiltonian Paths and Circuits

If $G = (V, E)$ is a graph with $|V| \geq 3$ then G has a Hamilton path / Cycle if there is a path / cycle in G that uses every vertex Once and Only Once.

Ex: Find a Hamilton path and Hamilton Cycle for W_5, if each exist.



a - e - f - d - c - b - a (Cycle)

a - e - f - d - c - b (Path)

conditions:

It follows that G contains a Hamilton Circuit, ... Path then G also contains a Hamilton ...

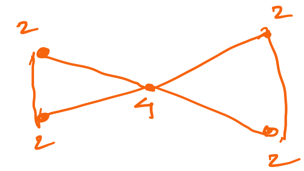
(by removing one edge in the circuit).

The converse is not necessarily true.

While there is no way for us to know when G, as a Hamilton circuit, we do have a few hints that will help us!

- If G has a Hamilton circuit, then
  $\forall v \in V,\ \deg(v) \geq 2$

- If $a \in V$ and $\deg(a) = 2$, the two edges incident with vertex a must appear in every Hamilton circuit for G.



- If $a \in V$ and $\deg(a) > 2$, then as we try to build a Hamilton circuit, once we pass through vertex a, any unused edges incident with a are deleted from further consideration.