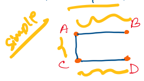
Lecture 7: Graph 1

Wednesday, April 22, 2026 11:27 PM

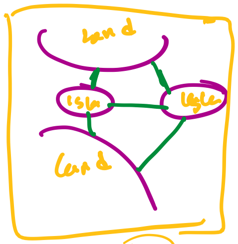
A graph $G = (V, E)$ consists of
V, a non-empty set of Vertices and
E, a set of Edges.

Edges connect either one or two vertices, called Endpoints.

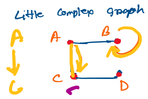


Simple graph!
- Each edge connects two different vertices
- No two edges connect the same two vertices.

AB
AC
CD



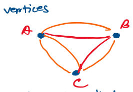
Little complex graph



B has a loop
Two edges connect A C

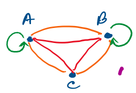
AC

Concepts! Multigraph
- Multiple edges connect the same two vertices

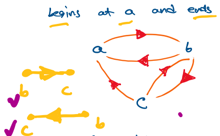


Loop! An edge that connects a vertex to itself

Pseudo graph! May include loops as well as multiple edges.



Directed graph - Digraph
A graph $G = (V, E)$ where each edge is associated with a pair of vertices.
Ex. The directed edge associated with (a, b) begins at a and ends at b.



(a, b)
(b, a)
(b, c)
(c, b)
(c, a)

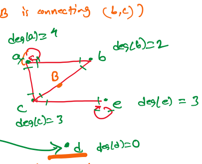
Summary of concepts!

Type	Directed Edge	Multiple Edge allowed?	Loops allowed?
Simple graph	NO	NO	NO
Multigraph	NO	YES	NO
Pseudo graph	NO	YES	YES
Simple directed graph	YES	NO	NO
Directed multigraph	YES	YES	YES
Mixed graph	BOTH	YES	YES

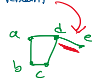
— x —

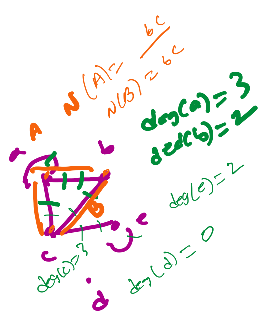
Unidirected Graph!

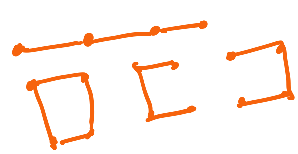
adjacent vertices (neighbors) (a, b)
Incident (Edge B is connecting (b, c))
neighborhood
$N(A) = \bigcup_{v \in A} N(v)$
Degree ! # of edg connected to vertices
Isolated
Pendant



$N(e) = \{a, b, c\}$

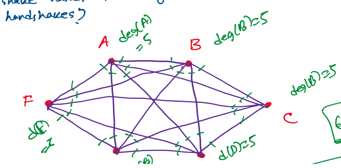






Handshaking Theorem

Suppose there are 6 people in a room, and each must shake hands with every other person. How many handshakes?



$\boxed{6 \times 5 = 30}$

$\frac{6}{2} \times (6-1) = 3 \times 5 = 15$

E D

$G = (V, E)$ with $\boxed{m}$ edges

$$\boxed{2m = \sum_{v \in V} \deg(v)}$$

$2m = 30$

$m = 15$ #

$$\frac{n(n-1)}{2} = \frac{6 \times \ldots}{2}$$

Ex: How many edges are there if you have 10 vertices each of degree 6?

$2m = 60$

$\boxed{m = 30}$

- Directed Graph:

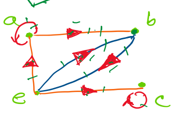
$\boxed{a \to b}$

adjacent to — a is adjacent to b

adjacent from — b is adjacent from a

Initial vertex — a

terminal / end vertex — b



	in-deg	out-deg
a	2	2
b	2	1
c	2	1
d	0	0
e	1	3
	7	7

# of edges = 7

• d

$\{ (a, a), \boxed{(a, b)} \ldots \}$

In-degree of vertex, $v = \deg^-(v)$ in-deg $(b) = 2$

out-degree of vertex, $v = \deg^+(v)$ out deg $(b) = 1$

$$\underbrace{\sum_{v \in V} \deg^-(v)}_{\text{sum of in-degree}} = \underbrace{\sum_{v \in V} \deg^+(v)}_{\text{sum of out-degree}} = \underbrace{|E|}_{\text{\# of Edges}}$$

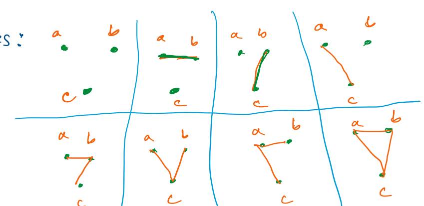
- Graph Isomorphisms

Ex. How many graphs exist with n vertices where the vertices are labeled?

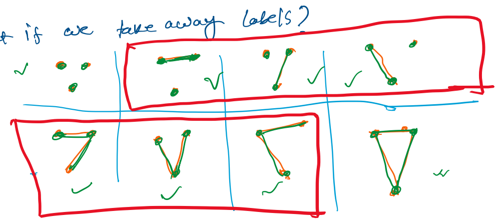
1 vertex: a •   ①

2 vertices: a •   b •   ②

3 vertices: ⑧



what if we take away labels?



Isomorphic

Two graphs are isomorphic if they have the exact same structural properties (# of vertices, edges, degree of vertices, etc).

Example: Are G and H isomorphic?

G H

To answer this, we need to know if we can relabel H such that each graph contains same vertices and the same edge sets.

Graph G : $V = \{1, 2, 3, 4\}$, $E = \{12, 34\}$

Graph H : $V = \{a, b, c, d\}$, $E = \{ad, bc\}$

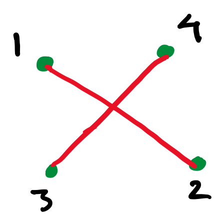
Now, can we relabel the graph H so that the vertex and edge sets match those of G ?

$f(1) = a$

$f(2) = d$

$f(3) = c$

$f(4) = b$

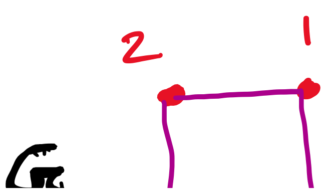


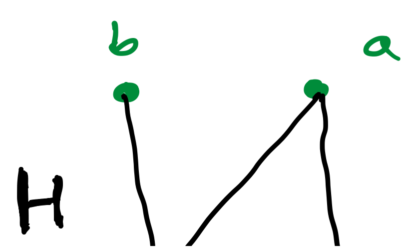
	1	2	3	4
1	0	1	0	0
2	1	0	0	0
3	0	0	0	1
4	0	0	1	0

	a	b	c	d
a	0	1	0	0
b	1	0	0	0
c	0	0	0	1
d	0	0	1	0

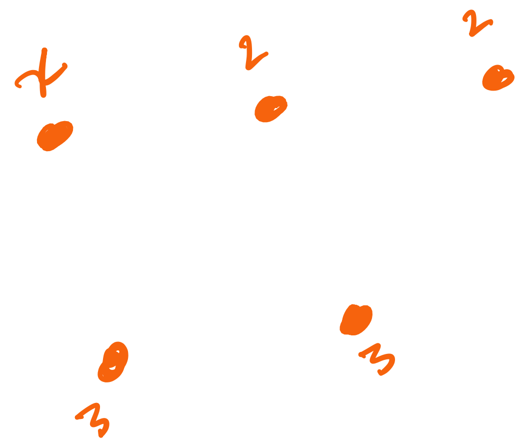
Ex! Is $G \cong H$? why or why not? if so, provide the mapping.

Symbol for Isomorphism






3
4
c
d


2
2
3
3