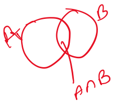
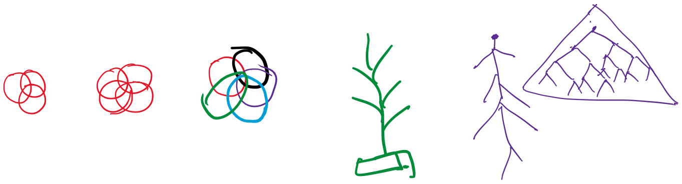
Lecture 6: Bayes' Theorem, Conditional Probability, Tree Diagram

April 16, 2026

When we are dealing w/ conditional probability, venn Diagrams may not be super helpful, Instead we can make a tree diagram that gives all the possibilities.





Review:

Let A an B are two events, with $P(B) > 0$. The conditional probability of A given B is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Ex: Among graduate students,

44% are binge drinkers;
37% are moderate drinkers; and
19% abstain from drinking.

Among those,

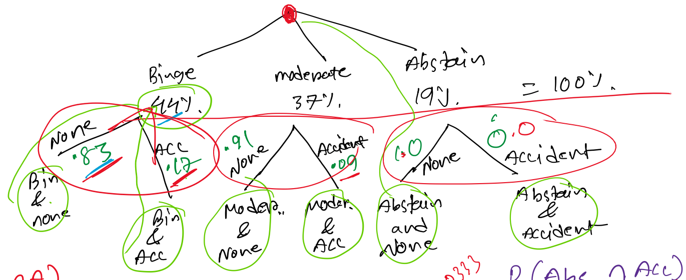
17% of binge drinkers have been involved in an alcohol related injury;
while 9% moderate drinkers have, and
0% of non-drinkers.

Draw a Tree diagram, then give the following:

(i) P(binge ∩ accident)

(ii) P(moderate ∩ no accident)

(iii) P(abstain ∩ accident)



$P(B \cap A)$
$= (0.44)(0.17)$
$= 0.0748$

$P(B \cap N) = (0.44)(0.83)$
$= 0.3652$

$P(M \cap A) = ?\ 0.0333$

$P(M \cap N) = ?\ 0.3367$

$P(Abs \cap Acc) = ?\ 0.0$

$P(Abs \cap N) = ?\ 0.19$

0.3652
0.3367
0.19
0.8919

—— x x x x ——

Q1: P(No accident) = ? $P(N) =$

Q2: P(Binge | no accident) = ?

$$\frac{P(B \cap N)}{P(N)} = \frac{0.3652}{??\ 0.8919} = 0.41?$$

—x—

Bayes' Theorem:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

w/ formula:

$$P(~~Binge~~ | \text{no accident}) = \frac{P(\text{no accident} | \text{Binge}) \cdot P(\text{Binge})}{P(\text{no accident})}$$

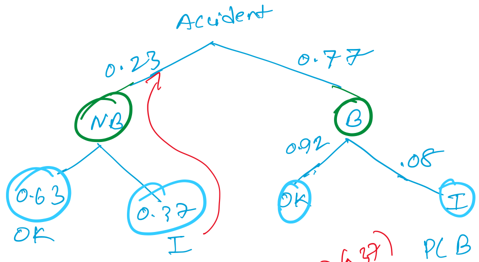
= ?

$$= \frac{(\overset{0.83}{??})(0.44)}{0.8919} = 0.41$$

In 77% of accidents, the driver was wearing

Example: a seat belt. For 92% of those drivers, there was no serious injury while 63% of non belted drivers escaped serious injury.

Make a tree diagram then find the probability a driver who was seriously injured wasn't wearing a seat belt.



✓ $P(NB \& I) = ? \; (0.23)(0.37)$ $P(B \text{ and } I) = ?$

$P(NB \& OK) = ?$ $P(B \text{ and } OK) = ?$

$$P(\text{no seatbelt} \mid \text{serious injury}) = \frac{P(NB \cap I)}{P(I)} = ?$$

w/ Baye's Theorem!

$$P(NB \mid I) = \frac{P(I \mid NB)P(NB)}{P(I)}$$

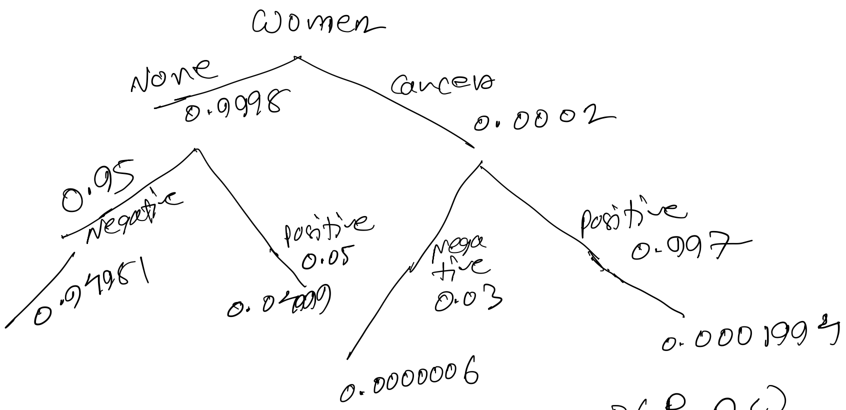
$$= \frac{(0.37)(0.23)}{(0.77)(0.08) + (0.23)(0.37)}$$

$$= 0.5842$$

Ex: Ovarian cancer afflicts 1 out of 5000 women. A new test is correctly detect ovarian cancer in 99.7% of women who have disease. However, it goes false positives to 5% of the women who do not have ovarian cancer.

Make a tree diagram and determine the woman who tests

probability that a
positive using this method actually
has ovarian cancer.



$$P(\text{cancer} \mid \text{positive}) = \frac{P(P \cap C)}{P(P)} = \frac{0.0001994}{0.0001994 + 0.04999}$$

$$= 0.00397$$

Exercise: At a local college, a survey of students
[redacted] who choose to live on campus
revealed 10% are seniors, 20% are juniors,
and the rest are underclassmen.
The most desirable dorm is the
"Gold" dorm, and 60% of seniors
elect to live there along with
15% juniors and 5% of freshmen/
sophomores.

what is the probability that a
randomly selected resident of the

"Gold" dorm is a senior?

Exercise!

Suppose a new test for coronavirus has been developed.

For people who have the virus, the test result is positive in 95% of the cases.

For people who do not have the virus, the test result is positive in 5% of the cases. These are known as false positives.

In a village, 1% of the population is actually infected with the virus. A person from the village is chosen at random and administered the test. The test result is positive.

Find the probability that the person actually has the coronavirus disease.