Lecture 5: Discrete probability

Tuesday, April 7, 2026 12:28 PM

Probability: Laplace definition

If S is a finite sample space of equally likely outcomes, and A is an event, where $A \in S$, the the probability of A is

$$P(A) = \frac{|A|}{|S|}$$

Q1: Find $P(6)$ for the experiment of rolling a dice?

Q_2: A bag of marbles contains

4 Green; 3 Red; 2 Blue

$P(\text{Green}) = ?$

$P(\text{Red}) = ?$

$P(\text{Blue}) = ?$

Q3: What is the probability that when two die are rolled, the sum of the dice is 3

$$P(3) = \frac{2}{6 \cdot 6}$$

Properties:

1. $P(S) = 1$
2. $0 \leq P(A) \leq 1$
3. $P(\bar{A}) = 1 - P(A)$ /complement Rule
4. $P(\text{at least 1}) = 1 - P(\text{none})$

   $P(\text{at least 1 H in 3 flips})$
5. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
6. $P(A \cap B) = P(A) \cdot P(B|A)$ (conditional probability)

Let S be a sample space of an experiment w/ a finite number of outcomes, we assign $P(s)$ to each outcome, s, so that

$$0 \leq P(s) \leq 1 \text{ for each } s \in S$$

and $\sum_{s \in S} P(s) = 1$

→ the function P from the set of all outcomes of the sample space S is called a probability distribution or probability model.

Ex: A coin is biased so that tails lands face up twice as often as heads. Find the probability distribution.

$$P(T) = 2P(H)$$

$1 = P(H) + P(T)$

$P(H) + 2P(H)$

$1 = 3P(H)$

$P(H) = \frac{1}{3}$

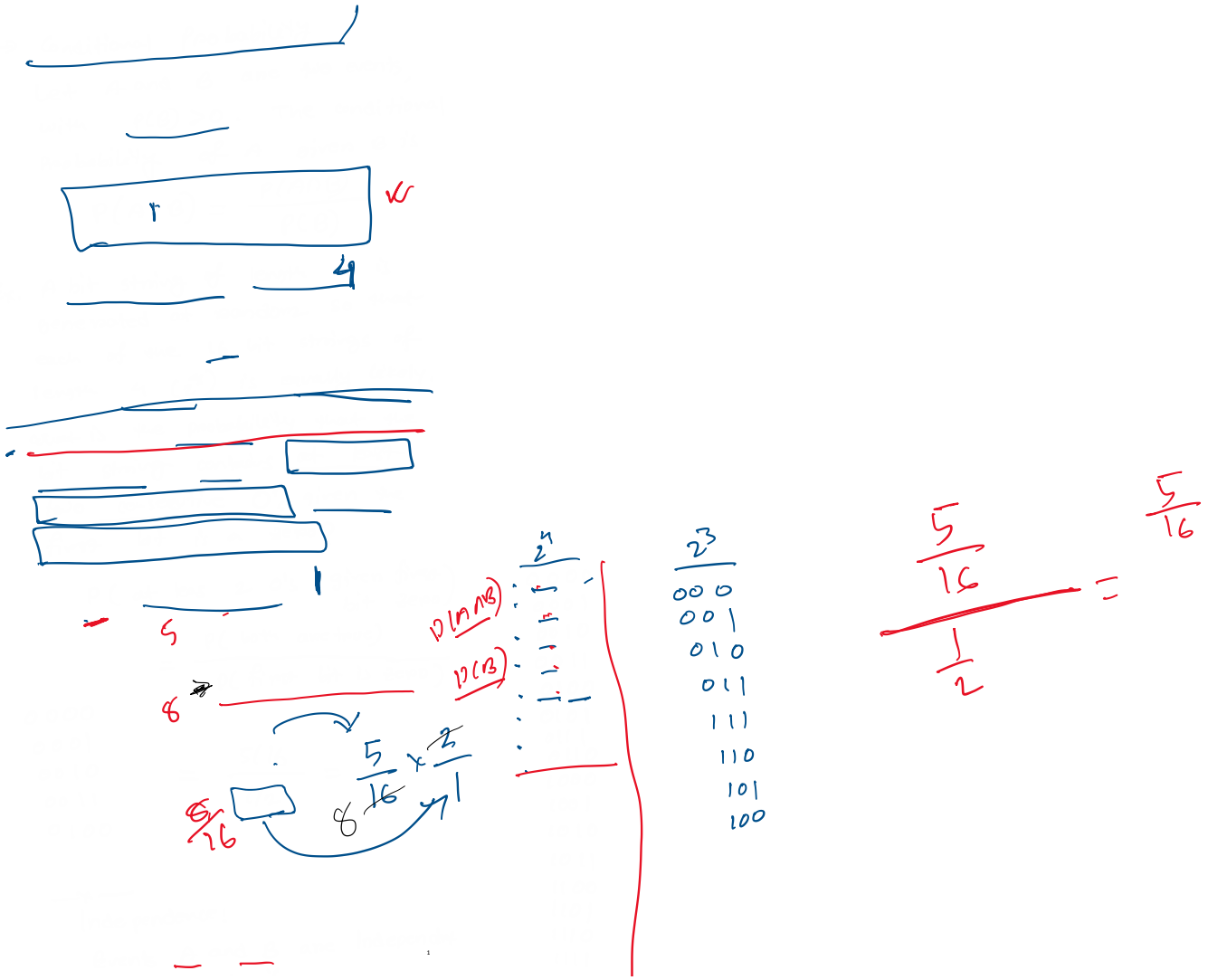
$P(T) = \frac{2}{3}$

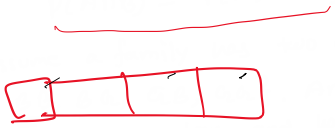
→ Union

If $E_0, E_1 \ldots E_n$ is a sequence of events (pairwise disjoint) in a sample space S, then

$= P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$ / $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ X

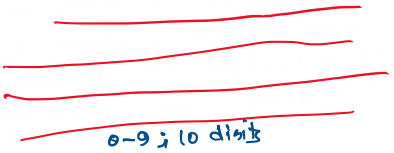
Suppose a dice is biased so that 2 appears twice as often as other 5 outcomes that are equally likely, what is P(even)?





BB
GB
BG

1/4
1/4 ≠



10 digits
5 times
10.10.10.10.10
10^5

$P(\text{winning}) = \frac{1}{10^5} = \frac{1}{100000} = 0.00001$

10.10.10.10.10
9 9/10 9
/10

$P(\text{at least 1}) = 1 - P(\text{None})$
$= 1 - (9/10)^5$
$= 1 - 0.590$
≈ 0.40

$P(A \cap B) = P(A) \cdot P(B|A)$

$\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} \cdot \frac{4}{49}$

$P(5,5) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5!}$

$5! \left(\frac{4^5}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}\right)$

≈ 0.000394

$4/52 \cdot 4/52 \cdot 4/52 \cdot 4/52$

≈ 0.000035