DSII: Final Exam

May 14, 2026

Name:	**ID:**

Exam Instructions

Show all relevant work.

- For an answer to receive credit, you must show the relevant calculation and justification for each response.
- Answers without supporting work may not be considered valid.
- Calculations must be shown for even multiple choice questions.

Academic integrity reminder. According to the UTEP Handbook of Operating Procedures, academic dishonesty includes committing or attempting to commit the following: **copying another student's work during an exam; using notes, books, or electronic devices during an exam without prior permission; taking an exam for another student; and communicating with or helping another student during an exam**.

1. A group contains 10 women, 8 left-handed people, and 3 left-handed women. How many people are either left-handed or a woman?

A. 15
B. 18
C. 21
D. 5

2. Which expression is mathematically equivalent to $\binom{12}{4}$?

A. $\binom{12}{8}$
B. $\binom{8}{4}$
C. $\binom{11}{4}$
D. $\binom{12}{6}$

3. A class of students takes a quiz that is worth 10 points. No partial credit is given, so every student receives an integer score from 0 to 10. What is the absolute minimum number of students needed to guarantee that at least two students receive the exact same score?

A. 10
B. 11
C. 12
D. 21

4. Assuming a calendar with 366 possible birthdays (to account for leap years), what is the minimum number of people required in a room to mathematically guarantee that at least two people share the exact same birthday?

A. 365
B. 366
C. 367
D. 400

5. To guarantee that at least 5 people in a randomly selected group share the exact same birth month, what is the minimum number of people required in the group?

A. 48
B. 49
C. 50
D. 60

6. Suppose the weights of people in a group are integers strictly between 100 lbs and 130 lbs. What is the minimum number of people needed to guarantee that at least two people weigh the exact same amount?

A. 30
B. 31
C. 32
D. 10

7. A coin is flipped five times. Event A is that the first three flips come up heads. Event B is that the last three flips come up tails. What is the exact probability of both events occurring simultaneously, $p(A \cap B)$?

A. 1/32
B. 1/8
C. 0
D. 1/4

8. A 5-card hand is dealt from a standard playing deck. Event A is that the hand has three aces. Event B is that the hand has a pair of kings. How many valid hands satisfy both Event A and Event B simultaneously ($|A \cap B|$)?

A. 0
B. 12
C. 24
D. 48

9. Suppose a fair coin is flipped three times. How many distinct outcomes are there in the sample space?

A. 2
B. 4
C. 6
D. 8

10. In an experiment where a coin is flipped three times, what is the size of the set corresponding to the event that at least two of the three flips come up heads?

A. 2
B. 3
C. 4
D. 6

11. A 5-card hand is dealt from a standard playing deck. Event A is that the hand has exactly three aces. Event B is that the hand has exactly three kings. What is the size of their intersection ($|A \cap B|$)?

A. 0
B. 1
C. 4
D. 24

12. A fair coin is flipped three times. Event F is that the first flip comes up heads. Event E is that the total number of heads is even (0 or 2). What is the probability of their intersection, $p(E \cap F)$?

A. 1/8
B. 1/4
C. 3/8
D. 1/2

13. How many positive integers strictly less than 100 have at least one digit that is a 9?

A. 9
B. 18
C. 19
D. 20

14. What is the coefficient of the x^4y^3 term in the expansion of $(3x - y)^7$?

A. $-3^4\binom{7}{3}$
B. $3^4\binom{7}{3}$
C. $\binom{7}{4}$
D. $-3^3\binom{7}{4}$

15. In an experiment consisting of a single roll of a fair red die and a fair blue die, what is the probability that the number on the red die is exactly one more than the number on the blue die?

A. 1/36
B. 1/12
C. 5/36
D. 1/6

16. A fair coin is flipped three times. Event A is that the first two flips come up heads. Event C is that the last two flips come up tails. What is the probability of their union, $p(A \cup C)$?

A. 1/4
B. 3/8
C. 1/2
D. 5/8

17. A loaded die is twice as likely to come up 6 than any of the other five possibilities. The distribution is defined by $p(1) = p(2) = p(3) = p(4) = p(5) = 1/7$ and $p(6) = 2/7$. If rolled twice, what is the probability that the first roll does NOT come up 6?

A. 1/7
B. 2/7
C. 5/7
D. 25/49

18. Using the same loaded die as above, the die is rolled five times independently. Let A_i be the event that the ith roll comes up at least 5 (5 or 6). What is the probability that all five rolls come up at least 5?

A. $(2/7)^5$
B. $(3/7)^5$
C. $(4/7)^5$
D. 3/7

19. What is the exact edge connectivity metric $\lambda(G)$ of the complete graph K_5?

A. 2
B. 3
C. 4
D. 5

20. In a standard deck of 52 cards across 4 suits, how many cards must be picked blindly to mathematically guarantee that at least 6 cards are from the exact same suit?

A. 20
B. 21
C. 24
D. 25

21. Three employees of a company, Anna, Fred, and Jose, have each worked on 12 projects. Each pair worked on four projects together, including one project that all three worked on as a team. What is the total number of distinct projects worked on?

22. How many integers in the range from 1 through 42 inclusive are divisible by 2, 3, or 7?

23. A team of three high jumpers all have a personal record that is at least 5 feet and less than 6 feet. Is it necessarily true that two of the team members must have personal records that are within four inches of each other? What if the team has four jumpers? (Assume heights are measured to within a precision of 1 inch.)

24. In an experiment consisting of three flips of a fair coin, what is the exact probability that the first two flips are both heads? Give your answer as a fraction simplified to its lowest terms.

25. A fair coin is flipped three times. Event A is that the first two flips come up heads. Event B is that the last two flips come up heads. What is the exact probability that the first two flips come up heads AND the last two flips come up heads ($p(A \cap B)$)?

26. A red die and a blue die are thrown. Both dice are fair. Event A is that the sum on the two dice is even. Event C is that the red die comes up 5. Calculate the conditional probability $p(A \mid C)$.

27. What is the exact coefficient of the x^2y^7 term in the binomial expansion of $(5x - y)^9$?

28. A red die and a blue die are thrown. What is the probability that the sum is strictly less than 10, given the condition that the two dice have the exact same number?

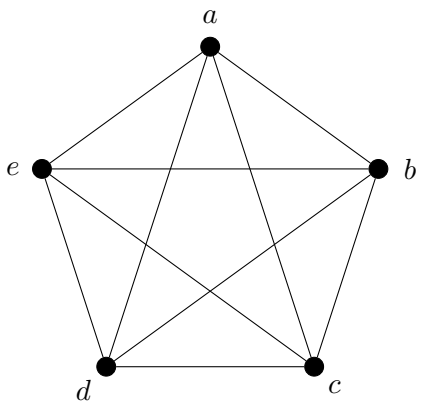
29. Ten kids are randomly grouped into an A team with five kids and a B team with five kids. Each grouping is equally likely. What is the size of the sample space for this experiment?

30. What is the exact vertex connectivity metric $\kappa(G)$ of the cycle graph C_{10}? Provide reasoning.

31. Using the inclusion-exclusion principle or counting by complement, calculate exactly how many 5-bit strings contain the sequence "100" as a consecutive substring.

32. Count the total number of strings of length 9 over the alphabet $\{a, b, c\}$ subject to the restriction that the first character is a OR the last character is a.

33. The undirected graph G with vertex set $\{a, b, c, d, e\}$ is shown below. Determine both the **vertex connectivity** $\kappa(G)$ and the **edge connectivity** $\lambda(G)$ of G. Fully justify your answers using the relevant definitions and theorems from graph connectivity.



34. Three copies of File 1 are stored at three distinct locations in a network of 30 computers, and three copies of File 2 are stored at three different locations in the same network. Six random computers fail. Formulate the exact probability that either File 1 or File 2 is completely wiped out.

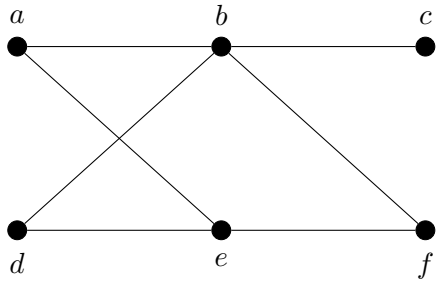
35. A class with n kids lines up for recess. The order in which the kids line up is random, with each ordering being equally likely. Two kids in the class are named Celia and Felicity. What is the probability that Celia is first in line AND Felicity is last in line? Simplify your final expression as much as possible so that the answer does not include any factorials.

36. A biased coin is flipped 10 times. In a single flip of the coin, the probability of heads is 1/3 and the probability of tails is 2/3. The outcomes of the coin flips are mutually independent. What is the exact probability that every single flip comes up heads?

37. The undirected graph G with vertex set $\{a, b, c, d, e, f\}$ and edge set

$$\big\{\{a, b\},\, \{b, c\},\, \{d, e\},\, \{e, f\},\, \{a, e\},\, \{b, d\},\, \{b, f\}\big\}$$

is shown below. Does G have a **Hamiltonian path**? If a Hamiltonian path exists, exhibit one explicitly. If no Hamiltonian path exists, prove that none can exist.



38. A graph has vertex set $V = \{a, b, c, d, e\}$. The degrees for vertices a, b, c, and d are $\deg(a) = 2$, $\deg(b) = 2$, $\deg(c) = 3$, and $\deg(d) = 3$. There are a total of 7 edges in the graph. What is the degree of the final vertex e?

39. A university offers three calculus classes: Math 2A, Math 2B, and Math 2C. A specific group of 157 students have all taken at least one of the three classes. Of these, 51 students have taken Math 2A, 80 have taken Math 2B, and 70 have taken Math 2C. Fifteen students have taken both Math 2A and 2B, 20 have taken both Math 2A and 2C, and 13 have taken both Math 2B and 2C. Using the Inclusion-Exclusion principle, calculate exactly how many students in this group have taken all three classes.

40. For which values of n does the complete graph K_n have an Euler circuit? Justify your answer based on the mathematical requirements of vertex degrees.